AFRL-WS-WP-TM-1999-9003

AN EMPIRICAL-THEORETICAL METHOD OF COMPARATIVE PREDICTION OF AIRPLANE PERFORMANCE

Air Service Information Circular, Volume I, No. 68

A. Branch

Air Service
Engineering Division
McCook Field
Dayton OH 45430

April 15, 1920

Approved for public release; Distribution unlimited.

AIR FORCE RESEARCH LABORATORY
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AIR FORCE BASE OH 45433

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
June 4, 1920

3. REPORT TYPE AND DATES COVERED
FINAL April 15, 1920

4. TITLE AND SUBTITLE
AN EMPIRICAL-THEORETICAL METHOD OF COMPARATIVE PREDICTION OF AIRPLANE PERFORMANCE
Air Service Information Circular, Volume I, No. 68

5. FUNDING NUMBERS

6. AUTHOR(S)
A. Branch

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Service
Engineering Division
McCook Field
Dayton OH 45430

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Service
Engineering Division
McCook Field
Dayton OH 45430

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFRL-WS-WP-TM-1999-9003

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; Distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
Historical study that highlights the advantages of using the empirical-theory method for airplance performance prediction.

14. SUBJECT TERMS
Velocity, Air Ceiling, Rate of Climb

15. NUMBER OF PAGES
14

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

File D 52.15/91

McCOOK FIELD SERIAL REPORT No. 1221

AIR SERVICE INFORMATION CIRCULAR

(HEAVIER-THAN-AIR)

PUBLISHED BY THE DIRECTOR OF AIR SERVICE, WASHINGTON, D. C.

Vol. I — June 4, 1920 — No. 68

AN EMPIRICAL-THEORETICAL METHOD OF COMPARATIVE PREDICTION OF AIRPLANE PERFORMANCE

(AIRPLANE SECTION, S. & A. BRANCH, A. D. M. No. 489)

Prepared by Engineering Division, Air Service
McCook Field, April 15, 1920

WASHINGTON
GOVERNMENT PRINTING OFFICE
1920

AN EMPIRICAL-THEORETICAL METHOD OF COMPARATIVE PREDICTION OF AIRPLANE PERFORMANCE.

I. RÉSUMÉ.

The present chart gives a complete picture of the variation of maximum velocity at any altitude and in the limit of the application indicates absolute ceiling. It is intended later to construct a similar chart for the determination of rate of climb at the ground. The rate of climb curve may be assumed to be a straight line; the present chart and the proposed one will indicate service ceiling and rate of climb at any altitude. Time of climb can be expressed graphically by means of a simple alignment chart. Three charts, then, will completely determine the performance of an airplane. In order that the completed chart may be available for use at once, this report will describe its derivation and use. The charts for rate of climb at the ground and time of climb will be published in report form as soon as they are completed.

The main variables entering into any consideration of ceiling and maximum speed at any altitude are pounds per horsepower, pounds per square foot of wing surface, and fineness, or lift/drift ratio. Several purely empirical charts have been presented in the past, and they are practically worthless because of the omission of the last-named variable. For instance, these charts would give for certain loadings per horsepower and per square foot identical performances for a large twin-engined bomber as for a small, cleanly designed pursuit airplane, whereas it can be shown that the L/D ratio of the latter may be 100 per cent greater than that of the bomber. Differences in high speed at the ground would be of the order of 25 per cent. Purely theoretical analyses of performance have been prepared with unsatisfactory results as a reward for long and tedious mathematical manipulations. The conclusion of a recent work of this nature emphasizes the need of more complete experimental data on (1) wings to determine the effects of various combinations; (2) parasite bodies, to determine the resistance of new shapes and the effect of interference; (3) propellers, to determine the thrust and torque; (4) engines, to determine drop in horsepower with altitude.

Obviously the only logical solution of this problem is an empirical-theoretical method. The inadequacy of the empirical method may be overcome by taking into consideration fineness, and the difficulties of the theoretical methods may be obviated by empirically solving the effects of wing combinations, interference, and propeller and engine performance at altitudes. An explanation of the preparation of the present chart will indicate to what extent the fundamental theories of flight have been incorporated.

II. PREPARATION OF CHART.

Maximum velocity attainable may be defined as that velocity at which the horsepower available is equal to the horsepower required. It is then only necessary to study the variation of these factors with altitude. Horsepower required at altitude depends on the fineness of the airplane and on the density. True velocity at altitude depends on density. These variations may be studied from a purely theoretical angle. Horsepower available at altitude depends on the engine-propeller group alone, and this variation is derived empirically. The presentation of all these variations is graphical and takes the form of a series of corrections to the pounds per horsepower scale, which is used indiscriminately to represent either #/H. P. available or required, since for any particular solution, they are equal.

Explanation of the chart will be expedited by reference to Figure I. Curves I of #/sq. ft. are first plotted against scale "a" of velocity and scale "b" of #/H. P., required, in this case. The basis of these curves are the wind tunnel results of a test on a model of the De H-4. Given the L/D and model lift curves, and solving the following equations through angles from near zero lift to maximum Ky, for various values of the total weight of the machine, we find different values of horsepower and hence #/H. P. required for different velocities and #/sq. ft. The area is kept constant throughout the various solutions.

$$V = \text{speed of test} \times \text{scale of model} \sqrt{\frac{\text{weight of airplane}}{\text{lift on model}}}$$

$$\#/\text{H. P.} = \frac{\text{Weight}}{\text{H. P. required}} = \frac{W}{\frac{DV}{375}} = \frac{375\ L/D}{V}$$

We shall next consider the effect of altitude on the disposition of the Curves I. At any angle of attack the velocity at altitude compared with the velocity at the ground will be—

$$V_a = \sqrt{\frac{1}{\delta}}\, V_o$$

and the horsepower required at this altitude and velocity will be—

$$HP_o = \sqrt{\delta}\ HP_a$$

where δ is the relative density at the altitude considered. Scale "b" of #/H. P. is now plotted along scale "c" so that if we start from scale "c" and go to the "ground" line of curves II, and then move over to scale "b," we would find the same #/H. P. as the one of scale "c." Starting again from a point on scale "c," going vertically to the 5,000-foot curve, thence horizontally to scale "b," we have multiplied the #/H. P. by $\sqrt{1/\delta}$, or 1.075. We next come to a certain #/sq. ft. on curves I which corresponds to a velocity V on scale "a." In order to find the true air speed at 5,000 feet, this velocity must be multiplied by $\sqrt{1/\delta} =$ 1.075, and the curves III effect this multiplication. High speed is now read on scale "d" instead of scale "a."

A further correction to #/H. P. required is next applied and takes into account the effect of fineness. The L/D model curves of the DH–4 were used as a basis and we will say that the fineness of the DH–4 is 100. H. P. required is directly proportional to L/D at any velocity. So that if we wish to use the DH–4 basic curves in finding the performance of an airplane of different fineness or relative L/D, we must decrease the #/H. P. required directly as this increase in L/D, or vice versa. The curves IV were next constructed and the #/H. P. scale moved from "c" to "e." Values marked on curves IV are not relative L/D ratios directly, but are the cube roots of the same. This is done simply to confine the values of fineness to narrower limits. Starting from a point, say, 10# per H. P. on scale "e," moving horizontally to the right to a fineness of, say, 110, we would find on scale "c" a #/H. P. of $10/110^3=7.5$. Moving vertically now to, say, 5,000 feet, we would find on scale "b" a #/H. P. of $7.5\times1.075=8.06$. The procedure from this point has been explained.

#/H. P. has now been corrected for fineness and altitude, and speed has been corrected for altitude. There remains only to correct #/H. P. available due to the variation of the engine and propeller performance with altitude. Scale "e" is first moved horizontally to scale "g." Scale "h" is an altitude scale and scale "f" is an engine-propeller factor scale. Suppose we have a specific engine curve VI. Starting on scale "h" at, say, 5,000 feet, moving to the right to curve VI, we would find on scale "f" a factor less than unity which would represent the horsepower available from the engine-propeller group in per cent of the horsepower available from the engine at the ground and with a propeller efficiency of 100 per cent. In this instance the factor would be about 0.66. This means that for a given performance (high speed) at 5,000 feet, the #/H. P. available at the ground would have to be increased by 1/.66 if the curves I are to be used. The curves V of #/H. P. available at the ground are so constructed that if we come from 5,000 feet on scale "h" to the engine curve and then up to, say, the 10#/H. P. curve, we should find on scale "e" a #/H. P. of $10/.66=15.2$. The procedure from scale "e" has been explained.

It must be remarked here that the assumption of constant fineness with altitude is not exactly true. It invariably decreases toward ceiling as maximum L/D is approached. The variation of this factor could be determined, but it would be included at the expense of making the chart much more complicated. Furthermore, any error arising through this assumption is modified to a negligible amount, since the effect of the variation is included in the engine curves VI. The fundamental purpose of the chart is a comparative prediction of performances, and once a general engine-propeller curve is established from full flight results, any effect of an error in assumptions is automatically eliminated by using this empirically derived curve. It is at this point that the difficulties of purely theoretical analyses are obviated or rather evaded. Again, the engine-propeller curve can be further broken up to take into account the variations of engine and propeller performance separately, but this can only be done after one or the other has been determined. Considerable work has been done at the Bureau of Standards on tests on the variation of engine power with altitude and if these are refined and become accurate, the proposed separation can be effected.

III. USES OF THE CHART.

Results of full-flight tests are used in conjunction with the chart for two purposes, to determine a series of engine-propeller curves and to establish a table of fineness factors for as many different airplanes as have been tested. Once these curves and a sufficiently long and varied table are available, it is proposed to choose for an airplane of unknown performance, but of known loadings, a suitable factor, and to them predict from the chart absolute ceiling and speeds at all altitudes. The proper engine-propeller curve must, of course, be used.

In Table I are presented a list of all airplanes on which complete full flight performance results are available, together with the fineness factor of each. The method for finding the fineness factor of any airplane is the following: Given a certain airplane, its weight, wing area, r. p. m. of the engine during high speed in ground tests, the maximum speed of the airplane and the horsepower of the engine corresponding to this r. p. m., the #/H. P. and #/sq. ft. are determined. Referring to the chart of Figure I, starting at point 1 corresponding to the given speed, move horizontally to the left to point 2 on the ground line of curves III, vertically to point 3, which is the given #/sq. ft., horizontally to the left to point 4 on the ground line of curves II. A vertical line 4–5 is drawn. Next, beginning on scale "h" at zero altitude, point 6, move horizontally to the right to point 7 of curves VI, vertically to point 8 corresponding to the given #/H. P., and draw the line 8–9. The intersection of this line with line 4–5 is point 10, which lies among curves IV and determines the fineness factor of the airplane in question.

It is to be noted that all of the engine-propeller curves correspond to a factor on scale "f" of 0.80 at the ground, which means that in determining fineness factors an arbitrary propeller efficiency of 80 per cent at the ground has been chosen. Since this is consistently done, and the fineness factor for a new airplane is selected by comparison with the airplanes of Table I, no error can arise from using 80 per cent rather than some other figure for propeller efficiency.

From flight test results at all altitudes the engine-propeller curves for the Liberty–12 and the Hispano–300 engines have been constructed. The general procedure in establishing an engine curve is the following: The #/H. P. based on r. p. m. corresponding to high speed at the ground is used throughout. Given the #/sq. ft. and the speed at all altitudes, the fineness factor is determined as previously outlined. Next, we shall determine a point on the engine-propeller curve at, say, 5,000 feet. Starting on scale "d" at point 11 corresponding to the high speed at 5,000 feet from full flight tests, move to the left to point 12 on the 5,000-foot curve, vertically to point 13 corresponding to the given #/sq. ft., to the right to point 14 on the 5,000-foot curve, vertically to point 15 on the fineness factor line previously determined, to the left to point 16 corresponding to the given #/H. P., and draw a vertical line 16–17. This line will intersect a line 18–19 drawn horizontally

from the 5,000-foot point on scale "h." The intersection point 20 determines the desired point on the engine-propeller curve. The same procedure is repeated for all altitudes at which full flight results are available. The engine-propeller curves of the Liberty-12 and the Hispano-300 were obtained by averaging curves obtained from performances of various airplanes equipped with these engines.

The use of the chart may now be extended to the prediction of the performance of an airplane of which is known the weight, the area, the engine, and the structural characteristics. By comparison with the airplanes of Table I, a fineness factor may be chosen, which is judged to be appropriate to the external proportions (wing truss, wing curve, chassis, fuselage), peculiar to the airplane under consideration. To find the speed at any altitude, it is only necessary to proceed through the chart from left to right, starting at the altitude scale "h," at, say, point 18, figure 1, and proceeding through points 20, 16 . . . 12, 11. If the curve of the particular engine used is not available, it will be necessary to select one of the given engine-propeller curves. As more full flight data is obtainable, engine-propeller curves for all engines in current use may be added to the chart.

As the altitude investigated approaches the ceiling of the airplane, point 13 moves upward along the particular #/sq. ft. curve until the apex of the curve is reached. This point must in general be approached by starting at points 18 corresponding to small increments of altitude, until an altitude is found at which the line 14–13 becomes tangent to the particular #/sq. ft. curve. Thus the chart serves a means of determining the absolute ceiling of an airplane.

An airplane may be built and ready for performance tests,and the engine-propeller curve of the engine used may have been previously determined and included on the chart. If the airplane is given a high-speed test near the ground, and the r. p. m. of the engine observed, then the fineness factor, the ceiling, and the speeds at all altitudes can be as accurately predicted from the chart as they can be by complete flight tests at all altitudes, and the work of flight testing can be considerably reduced.

A further use of the chart in conjunction with flight testing is suggested. For instance, if two complete performances are determined for a given airplane, engine, and pilot, but with different propellers, two engine-propeller curves can be established, and by direct division the relative efficiencies of the propellers can be determined at various altitudes or at the various corresponding V/nD ratios. Again, given two complete performances for a given airplane, pilot, and propeller, but with different engines, two engine-propeller curves are established, and by direct division the relative performances of the engines at various altitudes can be determined. In this connection, the increase of horsepower at any altitude due to supercharging can be accurately found.

Practical cases often arise in which it is desired to know the effect on the performance of an airplane, of changing from its present engine of certain weight and horsepower to another engine of different weight and horsepower. This change will generally result in a change in both the #/H. P. and the #/sq. ft. loadings, one decreasing and the other increasing. In these cases the charts present highly accurate results, quickly arrived at, whereas theoretical methods would entail a mass of computation and a considerable time.

Table I is especially valuable for the reason that the fineness factors are an exact indication of the relative worth of airplanes regardless of their loadings. For instance, the U. S. A. C-11 has a speed of 136 m. p. h. with loadings of 8.3 #/H. P. and 9.34 #/sq. ft., while the XB-1A has a speed of 124 m. p. h. with loadings of 9.6 #/H. P. and 7.38 #/sq. ft. An inspection of these speeds and loadings would serve no indication as to which is superior, and generally the U. S. A. C-11 would be chosen in preference to the XB-1A on account of the higher speed, but the table indicates clearly that if the two were reduced to the same basis—that is, loaded identically the same per horsepower and per square foot—the latter would be 4 per cent better than the former in regard to speed.

The chart has been extended to include very low horsepower loadings, high wing loadings, and fineness factors as high as 120. In view of this, it serves as an indication of the limits of speed performance which it is possible to expect when the horsepower loading of an airplane is reduced to a minimum dependent on weights per horsepower of engines, and when the loading per square foot has been increased to a point where landing speed, stability, or structural considerations limit it.

IV. CONCLUSION.

It can be shown that high speed at the ground is practically proportional to the cube root of L/D. The fineness factors of Table I, which are exactly the cube root of the relative L/D, are then a direct measure of the percentage of error which may be expected from using past charts of a purely empirical nature, based on #/h. p. and #/sq. ft. alone. The fineness factors of Table I vary from 91 to 116, so that the error above mentioned may be as high as 25 per cent.

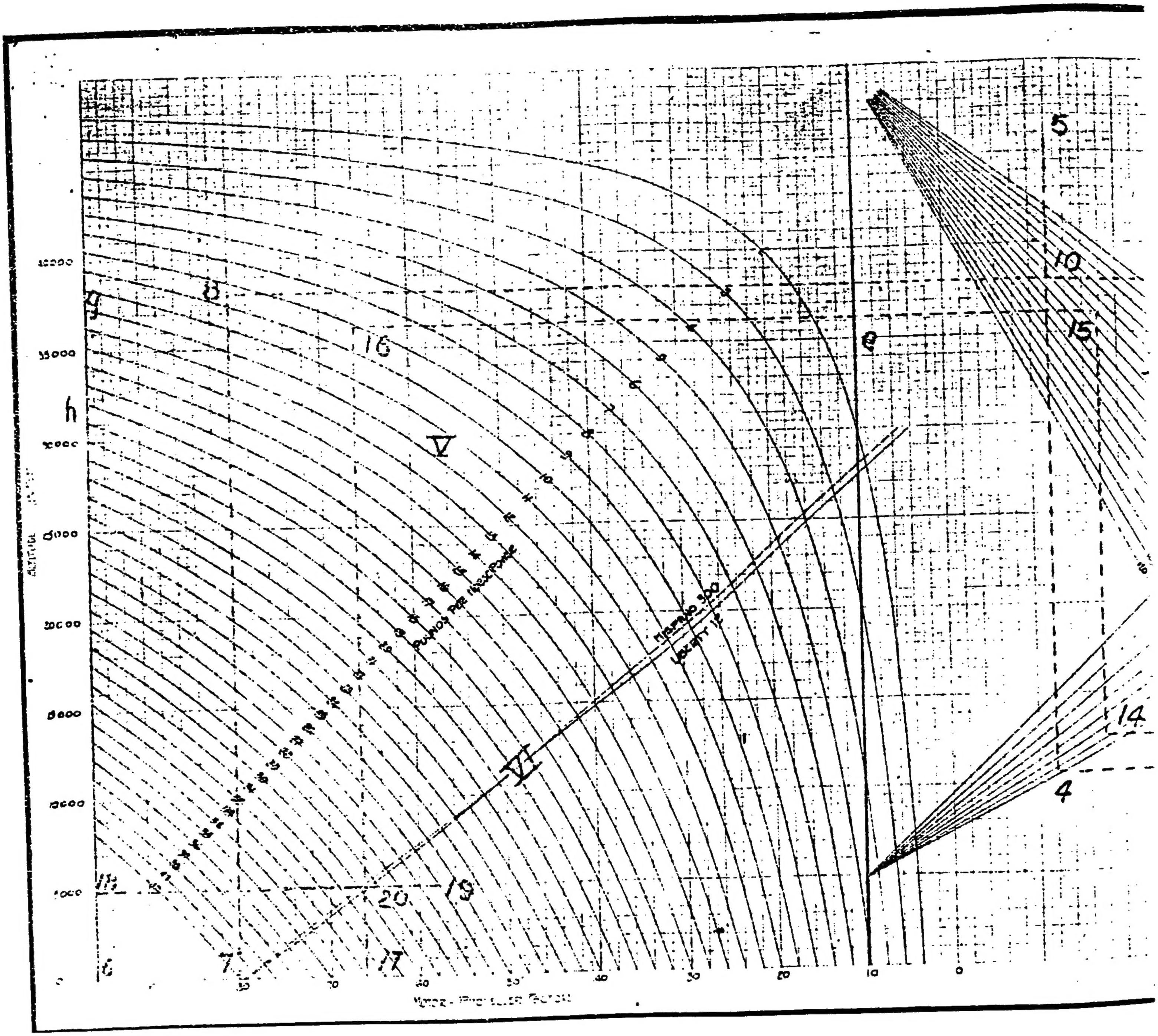
The author has done considerable work along the lines of purely theoretical analyses of performance, and is convinced that, due to the paucity of experimental data, the L/D of an airplane can not be computed with the accuracy with which it can be selected from Table I. As purely theoretical analyses are extended from the determination of the L/D curve to the computation of performance at altitude, errors and difficulties due to lack of experimental data on engine performance at altitude increase to such proportions that results are obtained so greatly at variance with full flight-test results as to be practically worthless.

It has been shown that the present empirical theoretical chart combines the advantages and avoids the difficulties of former purely empirical or purely theoretical methods of performance prediction, with a great saving of time over the latter and with a higher degree of accuracy than either. It must be remembered that as the volume of flight-test results increases so will the length and diversity of Table I increase, with the result that the exercise of proper judgment in selecting a fineness factor for a new airplane will continually be made easier and require less time.

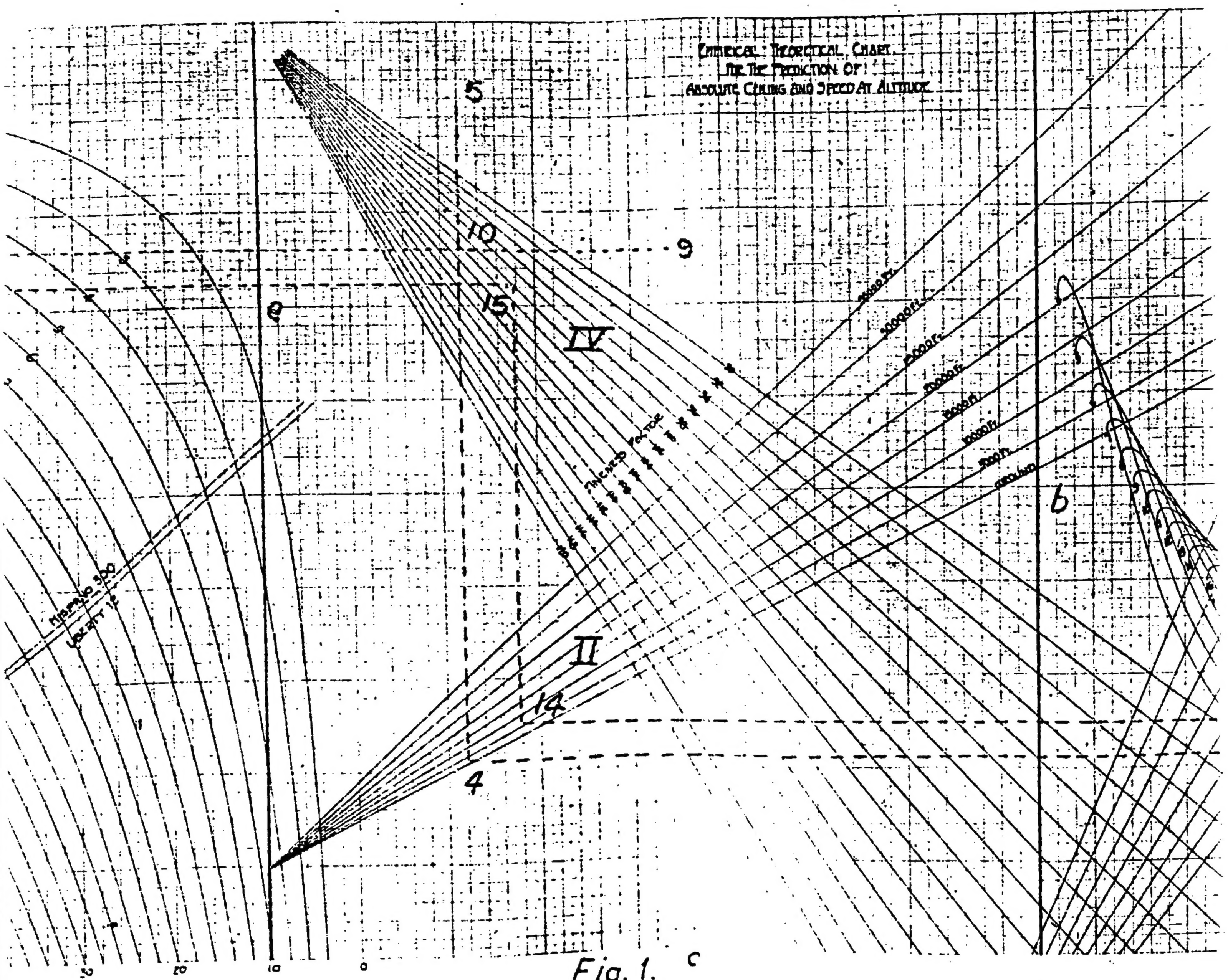
182136—20——2

Table I.

Airplane.	#/H. P.	#/sq. ft.	High speed at ground.	Fineness factor.	Airplane.	#/H. P.	#/sq. ft.	High speed at ground.	Fineness factor.
Friedrichshaven bomber	16.6	9.25	91	91	U. S. A. C-11	8.3	9.34	136	104
A. E. G. bomber	13.7	9.75	100	91	American SE-5	11.4	8.4	117	105
Le Pere triplane	9.75	9.85	114	91.5	VE-7	11.6	7.36	114	106
Martin bomber	12.3	9.57	104.8	92.5	Ordnance-300	7.05	9.3	147	107
Gotha bomber	18.7	9.50	89.5	93	XB-1A	9.6	7.38	124	108
Martin transport	12.3	9.57	106	93.5	Thomas-Morse MB-3	6.3	8.35	152	112
De H-4	9.8	8.9	120	100	SVA scout	8.4	8.5	143	114
Pomilio-12	11.1	7.88	111	100.5	Spad Herbemont	9.1	8.9	142	114.5
Fokker D-VII	10.8	8.53	117	102	VCP-1	8.72	9.72	150	116

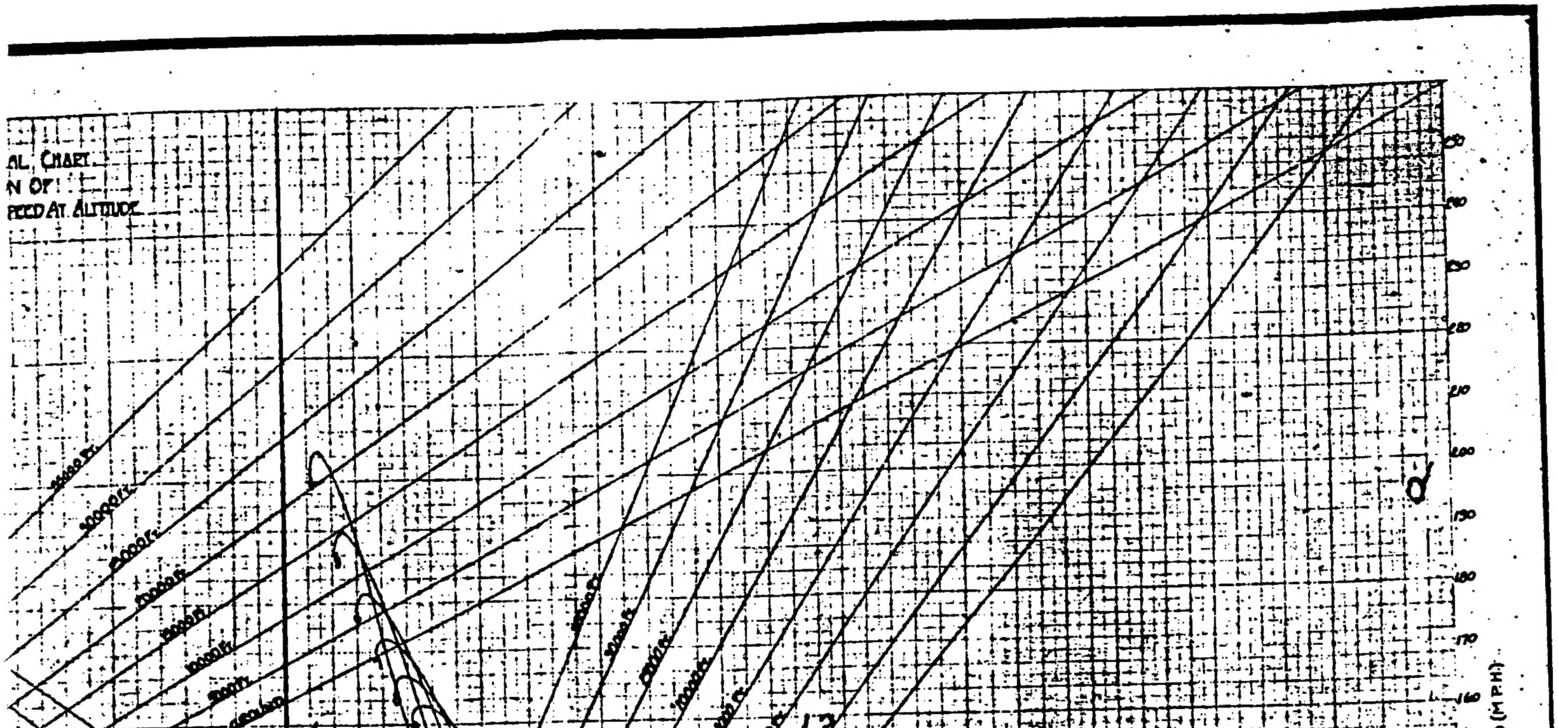


5
10
15
14
4
9
8
16
h
V
VI
19
20
17
7



EMPIRICAL-THEORETICAL CHART
FOR THE PREDICTION OF
ABSOLUTE CEILING AND SPEED AT ALTITUDE
5
10
9
15
8
IV
II
14
4
6
30
20
10
0

Fig. 1.



AL. CHART
N OF
PEED AT ALTITUDE
TRUE MAXIMUM SPEED (M.P.H.)
b
d
a
I
II
III
12
13

APPENDIX TO A. D. M. 489.

I. RATE OF CLIMB AT THE GROUND.

Rate of climb at the ground depends on the three variables, loading per horsepower, loading per square foot, and fineness factor, all of which have been taken into consideration in the determination of absolute ceiling and high speed near the ground from the chart of A. D. M. 489. For a given set of variables, the three above-mentioned characteristics of performance are definitely fixed. The criterion for determining rate of climb at the ground can be shown to be the product of absolute ceiling and high speed at the ground. Proceeding on this theory and using full flight results on the airplanes listed in Table II, the curve of figure 2 has been constructed.

The nature of the curve can readily be understood if the horsepower available and horsepower required curves of an ordinary velocity chart are borne in mind. The lower part of the curve is characteristic of the condition of high horsepower loadings and low fineness factors. In this case, an increase in horsepower available means an increase in rate of climb, but proportionately smaller increases in both absolute ceiling and high speed, so that the product of the latter two, if plotted against rate of climb, will give a curve of the nature of a straight line. The upper half of the curve corresponds to low loadings per horsepower and high fineness factors. In this case, the rate of climb and absolute ceiling are very nearly proportional to horsepower available, while high speed is proportional to the cube root of horsepower available. Thus rate of climb at the ground increases much faster than the product of absolute ceiling and high speed, and the nature of the upper half of the curve approaches parabolical nature, as might be expected.

II. TIME OF CLIMB.

Absolute ceiling and rate of climb at the ground being known, and assuming the rate of climb curve to be a straight line, the rate of climb at any altitude is known. Service ceiling is the altitude at which the rate of climb is 100 ft./min. The only remaining characteristic of performance is time of climb to any altitude. The Flight Test Branch has constructed an alignment chart for this purpose and it is reproduced in this report in figure 3. A vertical line, C–B, is drawn from the absolute ceiling, C, to any altitude B, and a horizontal line B–F is drawn to intersect line A–F. Point F is joined by a line F–G to point G on the rate of climb scale, where G is the rate of climb at the ground of an airplane having an absolute ceiling, C. Point H, where line F–G intersects the time scale, gives the time required for the airplane to climb to the altitude, B. Conversely, the altitude reached in a given time may be found.

The chart of A. D. M. 489 gives absolute ceiling and speed at all altitudes. Using in figure 2 the absolute ceiling and speed at the ground obtained from figure 1, the rate of climb at the ground is determined, and incidentally service ceiling and rate of climb at any altitude. Using in figure 3, the absolute ceiling obtained from figure 1 and the rate of climb at the ground obtained from figure 2, the time required to climb to any altitude is known. Thus, the performance of an airplane is completely, accurately, and quickly determined.

TABLE II.

Airplane.	Weight.	Area.	R.P.M. at ground.	H. P. at ground.	#/H.P.	#/sq. ft.	High speed at ground.	Fineness factor.	Absolute ceiling.	Rate of climb.
Martin bomber (with bombs)	10,225	1,070	1,665	832	12.3	9.57	104.8	92.5	12,300	630
Martin transport	10,225	1,070	1,665	832	12.3	9.57	106	93.5	12,600	660
Martin bomber (without bombs)	9,185	1,070	1,700	834	11.	8.56	106	94.	15,000	770
JN-4 D-2	2,016	352.5	1,456	90	22.6	5.72	73	97	9,250	335
DH-4	3,920	440	1,630	400	9.8	8.9	120	100	18,000	960
DH-9 bomber	4,372	490	1,750	425	11.5	9.95	116	100	14,800	860
Fokker D-VII	2,100	236	1,560	184	11.4	8.5	117	103	19,700	1,000
VE-7	2,095	284.5	1,725	180	11.6	7.36	114	106	19,400	975
Ordnance 300	2,432	261	1,885	345	7.05	9.3	147	107	23,300	1,460
SE-5	2,060	245.3	1,725	180	11.4	8.42	121.6	108	19,800	1,040
Thomas-Morse 300	2,095	252	1,810	333	6.3	8.35	152	112	25,500	1,920
XB-1A	2,994	405.6	1,875	345	8.7	7.38	133	113	23,600	1,300
VCP-1	2,600	269	2,000	360	7.22	9.72	154	114	24,500	1,680

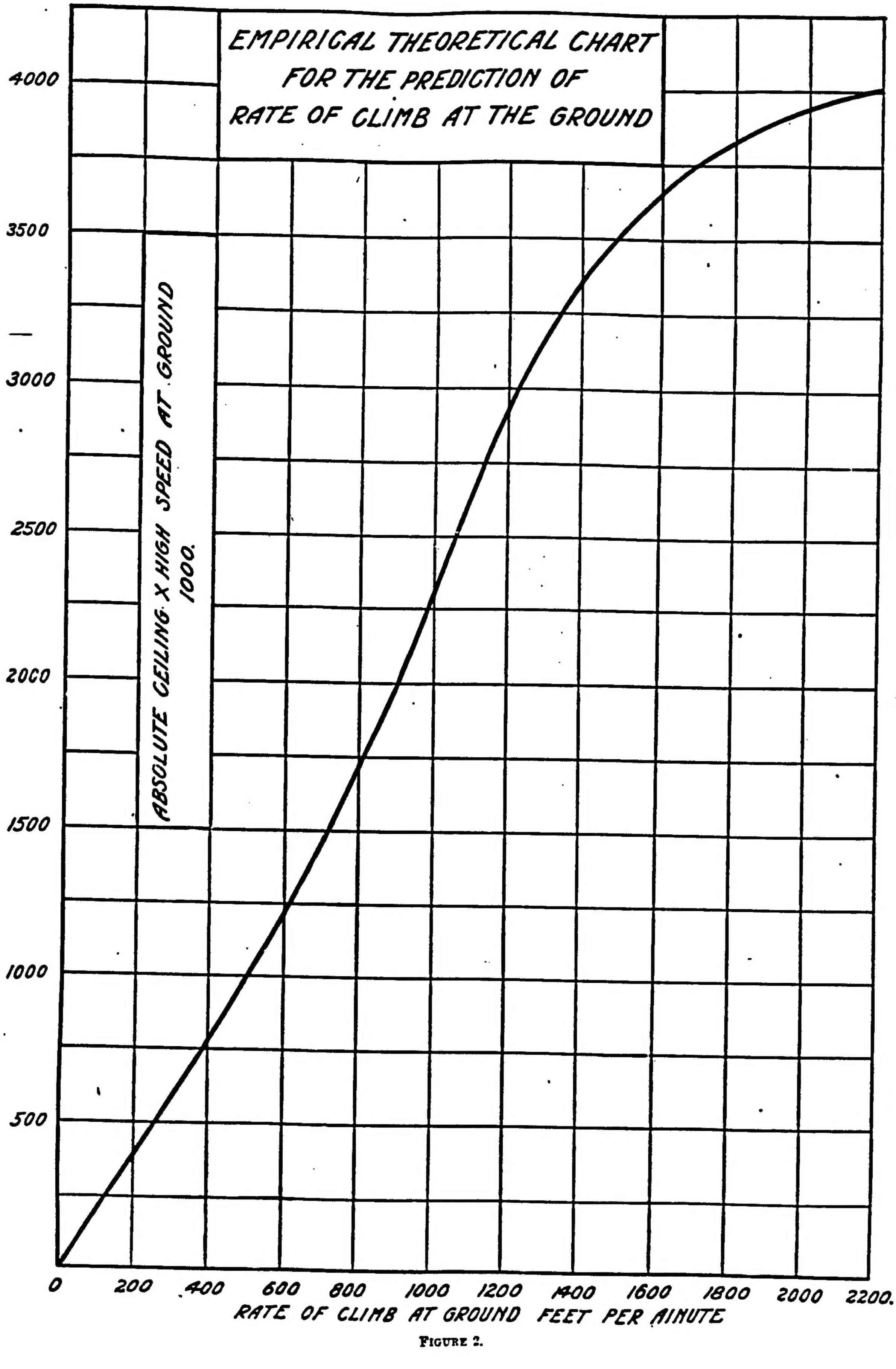

EMPIRICAL THEORETICAL CHART FOR THE PREDICTION OF RATE OF CLIMB AT THE GROUND
ABSOLUTE CEILING X HIGH SPEED AT GROUND / 1000.
4000
3500
3000
2500
2000
1500
1000
500
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200.
RATE OF CLIMB AT GROUND FEET PER MINUTE

FIGURE 2.

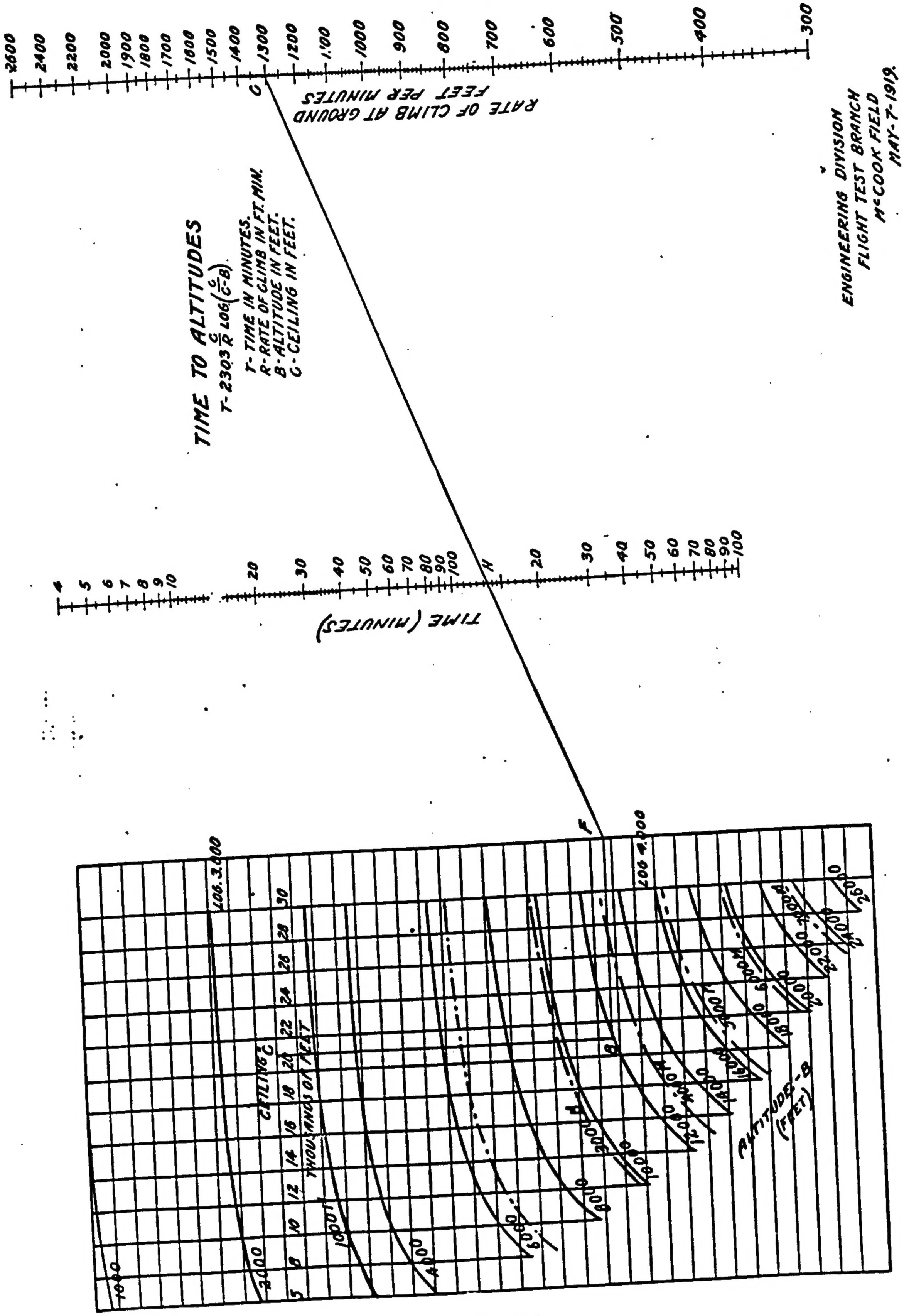

TIME TO ALTITUDES
T-2303 C/R LOG(C/(C-B))
T-TIME IN MINUTES.
R-RATE OF CLIMB IN FT. MIN.
B-ALTITUDE IN FEET.
C-CEILING IN FEET.
RATE OF CLIMB AT GROUND
FEET PER MINUTES
TIME (MINUTES)
CEILING C
THOUSANDS OF FEET
ALTITUDE-B
(FEET)
ENGINEERING DIVISION
FLIGHT TEST BRANCH
McCOOK FIELD
MAY-7-1919.

FIGURE 3.